

XXIV. *An account of observations made with the eight feet astronomical circle, at the Observatory of Trinity College, Dublin, since the beginning of the year 1818, for investigating the effects of parallax and aberration on the places of certain fixed stars; also the comparison of these with former observations for determining the effects of lunar nutation. By the Reverend JOHN BRINKLEY, D. D. F. R. S. and M. R. I. A. Andrews Professor of Astronomy in the University of Dublin.*

Read June 21, 1821.

THE results of the observations which I now beg leave to lay before the Royal Society, were instituted with a view of discovering, if possible, the source of the differences that have existed between the results of former observations made here, and of others made at the Royal Observatory at Greenwich; and they will, it is imagined, be found to be useful relative to some other important points in astronomy.

My former observations of certain stars pointed out a deviation of about one second from the mean place, after having made all the usual corrections. Mr. POND'S observations pointed out no such deviations. The deviations that I had found agreed with the effects of parallax. The observations that I have since made, far more numerous than the former, concur in exhibiting the same results: in showing deviations in certain stars that can be explained by parallax. Every other suggested solution of the difficulty appears quite inadequate

thereto. It is, I think, nearly demonstrated, that no change of figure in the instrument has occasioned it, and that the uncertainties of the changes of refraction can have had only a very small share, if any, in producing the effect observed.

It is not the results of a mere repetition of observations that I now offer to the Royal Society, but the results of numerous sets of such observations as seemed best adapted to examine the question in all its bearings. Some of them seemed particularly adapted to disprove, if wrong, the explanation by parallax.

All attempts to arrive at results inconsistent with parallax have failed ; so that, as far as the new observations are concerned, my former conclusions have been strengthened instead of weakened. I do not mean, however, to assert, that the subject is yet divested of the difficulties attendant on it from other sources. Some of the results that I have found, although in themselves in no manner inconsistent with parallax, will, justly perhaps with many, add to the difficulty of admitting the explanation by parallax. They will be unwilling to admit that many of the smaller stars are nearer to us than many of the brighter. That in a certain part of the heavens of considerable extent, many of the stars exhibit a sensible parallax. This however must be admitted, if my discordances result from parallax. If it be admitted, then several of the difficulties that have occurred by comparing my observations and those of Mr. POND, will be done away. But I shall defer a few remarks on this head, till I have given an account of my own observations, and of the results thereof.

The first set of results (Table 1) are from observations of

thirteen stars. These results contain the mean polar distance of each star reduced to January 1, 1819, the constant of aberration for each star, and the semi-parallax.

In deducing the quantity of parallax, the results must be affected by any uncertainty in the constant of aberration, since the times of the observations must necessarily be extended, so that the effects of aberration become sensible; and in like manner, in investigating the constant of aberration from observations of a given star, the parallax, if any, will be involved. Hence I adopted the following process in reducing the observations. The observed zenith distances of a given star were reduced to Jan. 1, 1819, by the common equations, taking the constant of aberration =  $20''\cdot25$ . The mean of these were taken. The correct mean zenith distance was supposed equal to this mean  $-e$ , the constant of aberration =  $20\cdot25 + x$ , and the semi-parallax =  $p$ . The equations of condition resulting from the respective observations thus contained three unknown quantities. These equations were reduced to three, by the method of making the sum of the squares of the errors a minimum. The solutions of these three equations give the values of  $e$ ,  $x$  and  $p$ , and thence the values of the mean polar distance, constant of aberration, and semi-parallax, as stated in Table 1.

In regard to the selection of these stars, some were selected with a reference to my former results as to parallax; others, as being convenient for the investigation of the constant of aberration.

The parallax resulting for  $\alpha$  Lyræ does not materially differ from my first determination. That of  $\alpha$  Aquilæ is less than

before. Had  $20\frac{1}{4}''$  been used for the constant of aberration, the result would have been only less by half a second than before.

In fact, the quantity of discordance does not differ from what I had before observed, but part of it now appears to arise from the constant of aberration being greater; a conclusion that will be deemed very important, should it be confirmed by future observations or other instruments. The parallax of Arcturus is somewhat less than before, and that of  $\alpha$  Cygni considerably less.  $\gamma$  Draconis, as before, exhibits no parallax; the small negative result of  $\frac{8}{1000}$  of a second may safely be referred to the unavoidable errors of observation. The new results agree with the former, in showing that the Pole Star has no sensible parallax.

With respect to the constant of aberration, it is almost unnecessary to remark its important bearing on the theory of light. Should a decided difference in the quantity of that constant, for two stars, be established, it would be decisive against the undulatory system; and it would also show, that the corpuscular theory could not, without the addition of principles at present unknown, explain the phenomena of light. I trust the results here obtained will be found to possess some interest, and may induce others to pursue the same object. I dare not venture to draw any conclusion from them relative to these important points. The two stars  $\eta$  Ursæ Majoris and  $\gamma$  Draconis appear to point out a difference. These stars, by their proximity to the zenith and other circumstances, are well adapted for obtaining exact results. The observations of each star seem to be very

good, as will appear by Tables 4 and 5. A continuation of observations will, I hope, enable me to speak with confidence as to the identity or diversity of these numbers.

The constant for  $\alpha$  Aquilæ will not be considered of so much weight as those of the higher stars, both on account of the more uncertain effects of refraction, and because only half the effect of aberration is visible in declination; although the influence of these circumstances is somewhat lessened by the greater number of observations.

The investigation of the constant of aberration by direct observations of zenith distance has not, that I am aware of, been attempted since those of BRADLEY, by the zenith sector. A century has nearly elapsed since his excellent observations were made. The results of M. DELAMBRE'S investigations, relative to the velocity of light, as deduced from the eclipses of Jupiter's satellites, appeared to confirm in so strong a manner the mean of BRADLEY'S results, that astronomers seem to have considered the point quite settled; but if I mistake not, one cause for this was the paucity of instruments adequate to so delicate an enquiry.

In considering the results with a view to the question of parallax, whether those that appear to point out parallax have not an origin in some cause unconnected with parallax: the first remark that offers itself is, that all the results furnish a positive parallax, if we except those small quantities in three of the stars which are quite within the limits of the unavoidable errors of observation. Might it not be expected that some of the stars would have furnished negative, as great as the positive quantities furnished by others? A considerable negative parallax would have been decisive. Again,

might it not have been expected that stars, in which the effect of parallax in declination is only a small part of the whole, would have shown a great parallax of declination as well as others, if the appearance of parallax is to be attributed to some other cause? Aldebaran,  $\beta$  Tauri,  $\alpha$  Orionis, Castor, Procyon, Pollux, &c. are so situate, that only a small part of the whole parallax could affect the declination, and therefore if these stars had exhibited a change of place of a second or two, it could not arise from parallax. The results of observations made with reference to this are given in Table 2. by which it will be seen that no sensible change of zenith distance takes place in these stars. This appears a very important circumstance. Also, these stars in summer passing the meridian in the day time, and in winter in the night, the absolute temperatures of the air differ much more than in the summer and winter passages of  $\alpha$  Lyræ and of  $\alpha$  Aquilæ; therefore naturally greater irregularities might be expected as to the former stars, than as to the latter. This also appears deserving of notice.

To examine this question in another way, I instituted a set of observations on stars in the same part of the heavens as those in which I had found the discordances that appeared to arise from parallax.  $\gamma$  Draconis I had already observed; and the circumstance of its not exhibiting the same changes of place, as I had found in  $\alpha$  Lyræ, appeared to afford a confirmation of my explanation. But this and  $\alpha$  Aquarii are the only stars out of seventeen that appear not to be affected by similar changes. Hence a new difficulty. It certainly is not likely that those stars, some of them only of the fourth magnitude, should be nearer to us than some of the stars of

the first and second magnitudes. The stars  $\gamma$  and  $\beta$  Aquilæ appear to have a parallax as great, or greater, than  $\alpha$  Aquilæ. The results of these observations are given in Table 3.

It is to be remarked, that these results cannot be considered nearly so exact as those of Table 1, because the observations are not nearly so numerous, and because the coefficients of  $p$  are in general much smaller. This latter circumstance could not be avoided in some of them, on account of their being too faint to observe in strong day light. For some of these stars also, the number of observations is so few, that a continuance of them may alter considerably the results; but with respect to others, this is not the case.

The value of  $p$  has not been deduced for  $\alpha$  Aquarii, because of the smallness of its coefficients; but as this star shows a much less discordance than the others, it would afford, as well as  $\gamma$  Draconis, an argument favorable for the explanation by parallax, were not its zenith distance so great, that some uncertainty with respect to refraction may take place.

The results, contained in the three first tables, have been deduced from so many observations, that it is impossible that the principal conclusions, although relative to such minute quantities, can be materially affected by the variable errors of observation. If error exist, it must be from some cause not to be controlled by mere observations. Two causes suggested themselves, which seemed to require particular consideration.

1. The instrument being in such different states as to temperature in summer and winter, may, by changing its

figure or otherwise, occasion the discordances of the zenith distances. If this were so, it must exist for all stars; and Table 2 shows satisfactorily it does not exist; for those observed when the difference of temperatures is greater than when  $\alpha$  Lyræ,  $\alpha$  Aquilæ, &c. were observed.

The same is deduced from the observations of the Pole Star. If the instrument give different results for the same angle, it must appear in the co-latitude determined by the Pole Star at different seasons. The co-latitude found by contemporaneous observations above and below the pole, is not affected by any uncertainty in the quantity of aberration, or in the parallax of the Pole Star; it therefore affords a good criterion of the permanency of the scale of measurement of the instrument, if I may so express myself. The quantities are as follow :

	No. of Observations.	Z. D. Pole Star.	Co-latitude.
Autumn {	72 76 S.P.	$\begin{array}{ccc} \circ & \text{ } & \text{ } \\ 34 & 57 & 21,24 \\ 38 & 16 & 11,84 \end{array}$	$\left. \begin{array}{ccc} \circ & \text{ } & \text{ } \\ 36 & 36 & 46,53 \end{array} \right\}$
Winter {	72 64 S.P.	$\begin{array}{ccc} 34 & 57 & 21,51 \\ 38 & 16 & 11,89 \end{array}$	$\left. \begin{array}{ccc} 36 & 36 & 46,70 \end{array} \right\}$
Spring {	64 71 S.P.	$\begin{array}{ccc} 34 & 57 & 21,26 \\ 38 & 16 & 11,71 \end{array}$	$\left. \begin{array}{ccc} 36 & 36 & 46,49 \end{array} \right\}$
Summer {	72 60 S.P.	$\begin{array}{ccc} 34 & 57 & 21,87 \\ 38 & 16 & 12,13 \end{array}$	$\left. \begin{array}{ccc} 36 & 36 & 47,00 \end{array} \right\}$

2. There may be an effect produced from the relative



temperatures of the external and internal air. The refractions have been computed by the internal thermometer. Now, at the summer observations of  $\alpha$  Aquilæ and  $\alpha$  Lyræ, &c. which take place between sunset and midnight, the external thermometer is oftentimes several degrees lower than the internal; the average is between  $4^{\circ}$  and  $5^{\circ}$ . At the winter observations, the external thermometer at the hours when these stars are observed, averages only about one or two degrees lower. Hence, if the refractions were computed by the external thermometer, the results as to  $\alpha$  Aquilæ and other stars of considerable zenith distance, would be less in favour of parallax.

But several circumstances induce me to conclude, that the true result is to be deduced from the internal thermometer. In a multitude of instances, were the external thermometer used, great discordances would take place. A great number of observations of circumpolar stars, made with a view to determine the constant of refraction, have given me nearly the same mean refraction as that determined by M. DELAMBRE from a great mass of observations of his own, and of M. PIAZZI, and which was also confirmed by the direct experiments of M. M. BIOT and ARAGO on the refractive force of air, whereas had I computed by the external thermometer, the constant of refraction would have been much less. Also I have found the mean zenith distance, computed by the internal thermometer, when it stood several degrees higher than the external, fully equal to that found when the external and internal thermometer stood at the same height. This has been particularly the case as to the Pole Star below the Pole. The circumstances of the results I have obtained by

this star seem to render it certain, that my instrument, and the mode of proceeding I have adopted, cannot lead to any material error. It is evident, that the constant of aberration determined by zenith distances of the Pole Star, when observed above the pole, should be the same as that determined from observations of the same star when below the pole. The same holds as to the parallax. A comparison of results will show the degree of accuracy that may be expected to be obtained. Now, by a reference to Table 1, it appears that the constants of aberration only differ by a very small fraction of a second, and the results for the parallax agree in showing it to be insensible for this star. The passages of the Pole Star being separated by twelve hours, the circumstances are in a manner reversed at the opposite seasons of the maxima of aberration and parallax.

The more this argument is considered, the greater weight it will, I think, be found to have. The object of our enquiry is to ascertain, whether the instrument measures exactly the interval between the two places of a star at the opposite seasons. We have two modes of doing it for the Pole Star under opposite circumstances, and we find the same result. It must however be admitted, that it is difficult to ascertain, with exactness, the consequences of the differences of external and internal temperatures. It is a matter of some importance, and I hope to be able to make farther observations for ascertaining, more exactly, its bearing on the present question. In the mean time I beg to state distinctly, that, after reviewing all the circumstances of my observations, I do not consider my conclusions materially affected on this account.

Mr. POND mentions, that in winter he endeavoured to

equalize the internal and external temperatures. Here the difference of temperatures is greatest after sunset in summer and autumn, except in extreme cold in winter ; and the equalization of the temperatures cannot be easily affected without too great an exposure of the instrument to the external air. Partial currents might derange it, and occasion more uncertainty than that arising from the difference of temperatures. The room in which my instrument is placed, containing also the transit instrument, is of considerable dimensions, being thirty-seven feet long, twenty-three feet broad, and twenty-one feet high. The instrument is several feet from the shutters, which may be supposed a favourable circumstance. The apertures for observation are three feet wide.

Having thus given a detailed account of observations that have been principally instituted with a view of obtaining an explanation of the source of the difference of the results of my former observations and of those of Mr. POND, relative to parallax ; it is with concern I state, that it contains not a trace of *any such explanation*. I have been unable to obtain *any result* that is *opposed* to my former conclusions.

It would be extremely important to ascertain the certainty of the results of an instrument, which, by its construction and principle of reversion, seems much better adapted to the present wants of astronomy than a mural circle. The advantage of referring each star to the apparent zenith point, and thus obtaining a knowledge of its motions without a reference to those of other stars, is easily appreciated. The advantage is also very great, of being able to observe a few minutes before the object arrives at the meridian, and, reversing the instrument, of then observing again. The zenith distance is thus

obtained completely, without a reference to the correction for collimation; and we are not obliged to depend for some days, perhaps, on the stability of the correction for collimation. We also are more likely, in this way, to improve our theory of refraction, because thus the irregular refractions of different days will not be mixed together. These considerations, independently of the interest of the question of parallax and aberration, lead me to dwell more on the discordance of the Greenwich observations, and of those made here, than otherwise I should be willing to do: and I am induced to offer a few brief remarks relative to the circumstances of the observations that have been adduced by Mr. POND, to prove the non-existence of a visible parallax.

1. The observations of the Greenwich mural circle are so implicated with each other, and the polar distances, even of the high stars, depend so much on the index error obtained by observations of those stars in which the uncertainties of refraction and of other data produce their effects, that it is not very extraordinary that the small quantities which I ascribe to parallax should not *distinctly* appear from the observations of the mural circle. There is indeed one exception to this explanation, which, I freely confess, occasions in my mind more difficulty than any other. This is in regard to  $\gamma$  Draconis and  $\alpha$  Lyræ. According to the observations of Mr. POND, there is no difference between the relative places of these stars in summer and winter; and it is from a relative change of place I find in these two stars, that I adduce, what appears one of my strongest arguments for the parallax of  $\alpha$  Lyræ. In this instance, the two instruments are completely at variance, and one of them must give an erroneous result.

2. The fixed telescope, used by Mr. POND for the comparison of  $\alpha$  Cygni and  $\beta$  Aurigæ, shows no relative changes of place that can be explained by attributing a parallax to  $\alpha$  Cygni. This star formerly appeared to have a less parallax than others I had observed. My new observations give a much smaller quantity for it; but I am inclined to think the true quantity lies between my present and former results. Now admitting it to be half a second, no contradiction to this can be drawn from the observations by the fixed telescope, when those observations are carefully examined with a reference to the visible effects of the change of temperature

The fixed telescope used for  $\alpha$  Aquilæ made the comparison by  $\zeta$  Pegasi. Now, the same maxima of parallax in declination of this star and of  $\alpha$  Aquilæ occur within a few days of each other, so that it is completely the difference of parallax that is ascertained by comparing this star and  $\alpha$  Aquilæ; and my results in Table 3 show, that in this part of the heavens we cannot conclude any thing as to the absolute parallax of one star by its relative parallax to that of another. Thus I cannot but venture an opinion, that nothing certain has hitherto been determined by the use of the fixed telescopes.

3. The results of the investigation of the parallax of  $\alpha$  Aquilæ, by observations in right ascension, are still less satisfactory. The stars Mr. POND has principally used for determining the error of the clock, are those in which I find the principal discordances, as will appear by a reference to the Greenwich observations; and consequently, those results ought to afford no appearance of parallax.

If stars opposite in right ascension be used, the utmost

exactness as to the stability and construction of the transit instrument and uniformity of the rate of the clock, is required. The Greenwich transit may be considered fully adequate; but it is evident the clock is not so perfect as it ought to be. In order to avail ourselves of this method, by stars opposite in  $\mathcal{R}$ , at first view so plausible, of examining the question of parallax, skies much less changeable than those we are accustomed to will be required. As to this observatory, it rarely happens that a cloudless sky continues for twenty-four hours together. The entire of the observations from which my conclusions have been deduced will, I hope, soon be published. The particular results, therefore, of part only, are here added, that the nature of the observations, and the accuracy to be expected from them, may be more fully understood.

In Tables 4 and 5 will be seen the errors of each observation of  $\eta$  Ursæ Majoris and of  $\gamma$  Draconis, assuming as exact the results of all the observations of each star as to the mean zenith distance, parallax, and constant of aberration. These two stars have been chosen as examples, because in these the constants of aberration differ more than in other high stars. These stars being so near the zenith were observed on the meridian.

*Of the 99 observations of  $\eta$  Ursæ Majoris.*

2	In 2 observations the error exceeds 2"
18	In 20 observations the error exceeds 1
79	In 79 observations the error is under 1
99	

*Of the 152 observations of  $\gamma$  Draconis.*

5	In 5 observations the error exceeds 2"
48	In 53 observations the error exceeds 1
99	In 99 observations the error is below 1
152	

The errors of each observation of  $\alpha$  Lyræ and of  $\alpha$  Aquilæ are also given in Tables 6 and 7, adopting the results from all the observations of mean zenith distance, parallax, and constant of aberration, as exact. These stars have been selected on account of the great parallax deduced.

*Of the 157 observations of  $\alpha$  Lyræ.*

2	In 2 the error exceeds 3"
6	In 8 the error exceeds 2
44	In 52 the error exceeds 1
105	In 105 the error is below 1
157	

*Of the 135 observations of  $\alpha$  Aquilæ.*

2	In 2 the error exceeds 3"
8	In 10 the error exceeds 2
41	In 51 the error exceeds 1
84	In 84 the error is under 1
135	

In the above, the results of the observations of one day are considered as a single observation; but in computing the values of  $e$ ,  $x$ , and  $p$ , each result was considered as having a weight proportional to the number of observations; that is,

each bisection of the star and reading off was considered as a distinct observation. The great improvement in the uniformity of the results is very apparent when two or four observations are made on the same day, by observing before and after the object has been on the meridian.

The greater errors occur according as the star is more remote from the zenith. This is doubtless occasioned by the irregularity of refraction, which is so very apparent when the object is within  $10^{\circ}$  or  $15^{\circ}$  of the horizon. It may be traced by my observations to within a few degrees of the zenith. On this account, when the object is  $40^{\circ}$  or  $50^{\circ}$  from the zenith, and great exactness is required, it will be necessary to increase the number of days of observation, rather than the number in the same day, that the irregularity may disappear from the mean.

I know not of any observations where the irregularity of refraction appears so distinctly as in mine. To illustrate this more fully, I have, in Table 8, added the observations of  $\alpha$  Aquarii. This star I observed with a view of ascertaining whether it was subject to changes of place similar to what appeared in  $\alpha$  Aquilæ. The mean results give a much less change of place, but the discordances which appear to belong to refraction are more fully apparent.

By observing before the star came to the meridian, and then reversing the instrument, using only the bottom microscope, I was enabled to get several results on the same day. In all the other stars three microscopes were used.

An inspection of Table 8 appears to show clearly the effects of this irregular refraction. Thus it is evident, that the differences between the results of the observations of



December 16, 1818, and December 28, 1820, must have been occasioned by the irregularities of refraction, as the respective observations of each day, in both positions of the face of the circle, are very consistent with one another. The same remark may be made as to the observations of August 17, 1819, and of September 6, 1820, &c. &c.

An illustration of the method of observing, &c. is given from  $\gamma$  and  $\beta$  Aquilæ in Tables 9 and 10.

The earlier observations of these stars were made on the meridian, and then the mean of the three microscopes, the refraction, and the mean zenith distance, January 1, 1819, as deduced from each observation, are given. Afterwards, when the observations were made off the meridian, the sidereal time elapsed between the observation and the passage over the meridian, is also given. The coefficients of  $x$  and  $p$  for each observation are also given.

In regard to the reductions of the observed zenith distance to the mean zenith distance: the precessions in N. P. D. corrected for proper motion, as given in the Nautical Almanac, were used. These annual variations agree so nearly with the annual variations deduced by using Mr. BESSEL's precessions, and the proper motions deduced by a comparison of Mr. BESSEL's results from Dr. BRADLEY's observations with the modern observations, that no inexactness can arise on this account.

The equation, in polar distance, used for lunar nutation was  $-8'', 28 \sin (\mathcal{R} - \mathcal{Q}) - 1'', 22 \sin (\mathcal{R} + \mathcal{Q})$

By a comparison of my observations of certain stars made 1809, 1814, and those made lately, I find this equation of lunar nutation  $-8'', 06 \sin (\mathcal{R} - \mathcal{Q}) - 1'', 19 \sin (\mathcal{R} + \mathcal{Q})$

If this should turn out, as I believe it will, more exact than the former, it will occasion no difference of results as to parallax and the constant of aberration.

The solar nutation I used was  $-0''$ ,  $48 \sin(\mathcal{R} - 2 \odot)$ , not regarding the smaller term. With my lunar nutation, the solar nutation will be  $= -0''$ ,  $52 \sin(\mathcal{R} - 2 \odot) - 0,02 \sin(\mathcal{R} + 2 \odot)$ . That which I used, therefore, is sufficiently exact.

The small terms depending on  $2$  long. moon, have not been noticed on account both of their smallness and of the quickness of their period. The principal term of the nutation in North Polar distance depending on  $2$  long. of moon  $= -0''$ ,  $08 \sin(\mathcal{R} - 2 \text{D})$ ,\* which going through its period in the short space of a fortnight, can occasion no error in the results that I have obtained.

To the stars above given, for which the constants of aberration have been investigated, may be added  $\alpha$  Cassiopeæ,  $\alpha$  and  $\beta$  Cephei. The observations relative to parallax for these stars have not been sufficiently numerous to use the method of least squares.

\* This term was stated in my paper in the Philosophical Transactions, 1818, as being  $= -0,04 \sin(\mathcal{R} - 2 \text{D})$ . I had adopted the numbers in the *Mec. Cel.* Tom. 2, p. 350, for the coefficients of  $\sin 2 \nu'$  and  $\cos 2 \nu'$ , but on examination I found that M. LAPLACE had omitted to multiply by  $\lambda(3)$ .

I may also remark, that in my two former papers on this subject, I unnecessarily, and in the first erroneously, introduced the effect of the elliptical motion of the earth in computing the aberration. The aberration in N. P. D. computed by the formula  $m \cos(\odot \sim \mathcal{K})$  differs from the true quantity by the constant quantity  $\frac{1}{60} m \cos(9^s. 9^o. \sim \mathcal{K})$  and therefore the mean place of the star needs only to be regarded.

	No. of Ob.	N. P. D. Jan. 1, 1819.	Constant of aberration.
$\alpha$ Cassiopeæ	87	$34^{\circ} 27' 23.47''$	$20,30—,14p$
$\alpha$ Cephei	100	$28 10 42.74$	$20,56—,39p$
$\beta$ Cephei	62	$20 13 57.05$	$20,20—,38p$

In my paper, Philosophical Transactions 1819, the constant for  $\alpha$  Ursæ Majoris was given. In Table 1,  $\beta$  Ursæ Majoris has been introduced; both could not be observed at the same time; and having formerly intended to deduce the constant of aberration from the mean of a great number of stars,  $\beta$  Ursæ Majoris was observed.

The importance of the enquiry relative to the velocity of light, has since induced me to multiply as much as possible observations of the same star, and therefore the observations of  $\alpha$  Ursæ Majoris have not been resumed.

#### *Lunar Nutation.*

A comparison of zenith distances of certain of the stars that I observed in the years 1809-1814, and of the zenith distances of the same stars observed in the years 1818-1820, has given the following results relative to lunar nutation.

It is almost unnecessary to remark, that those stars only were used in which the nutation at each period was nearly a maximum, with contrary signs. Two circumstances are particularly required to obtain the most accurate results.

1. That a comparison of results should be deduced from observations made by the *same* instrument.

2. That observations should be continued through a whole period of the lunar nodes, in order to ascertain, with exactness, the annual variation of zenith distances for each star.

The latter condition can only be fulfilled hereafter for my instrument. In the mean time, no material uncertainty can arise from the want thereof. The accurate reductions of Dr. BRADLEY'S observations by Mr. BESSEL, have given us, with much exactness, the mean N. polar distances in 1755 of the stars I have used. Three periods nearly of the lunar nodes intervened between 1755 and my former observations. Hence, assuming the change from precession, as deduced by Mr. BESSEL, the proper motion of each star was obtained; this proper motion was then applied to the precession of each star for the years 1815 and 1816, which was also deduced by help of Mr. BESSEL'S precessions. The annual variation of each star, thus obtained, for the middle of the interval between my two sets of observations, was used in connecting those sets to determine the exact effect of lunar nutation.

As the lunar nutation in N. P. D. used was,

$$- 8'', 28 \sin (\mathcal{R} - \mathcal{Q}) - 1'', 22 \sin (\mathcal{R} + \mathcal{Q})$$

and therefore the greatest term of the nutation of obliquity of ecliptic =  $9'', 50 \cos \mathcal{Q}$ ; I supposed the true coefficient of this latter =  $9'', 50 (1 + y)$  and then found as follow:

	Number of Observations in 1808-1814.	Number of Observations in 1818-1820.	Equations deduced.	Greatest coeff. of Nutation of Ob. Eclip.
Capella	30	96	" 54,20 + 8,49y = 53,50 - 7,79y	" 9,09
$\beta$ Tauri	18	84	21,73 + 8,66y = 21,65 - 7,63y	9,45
$\alpha$ Orionis	18	148	9,24 + 8,81y = 7,98 - 7,09y	8,75
Castor	10	66	30,23 + 8,92y = 30,42 - 5,18y	9,62
Procyon	16	136	8,41 + 8,91y = 7,30 - 4,88y	8,74
Pollux	10	65	44,98 + 8,79y = 44,29 - 4,81y	9,01
$\gamma$ Draconis	27	132	7,54 - 8,65y = 7,90 + 5,80y	9,26
$\alpha$ Lyræ	126	155	42,69 - 9,14y = 42,94 + 7,89y	9,36
$\alpha$ Aquilæ	76	238	4,94 - 8,74y = 5,10 + 7,42y	9,40
$\alpha$ Cygni	47	120	42,15 - 7,48y = 42,77 + 4,97y	9,03
	378	1240		9,25

On account of the small number of observations of some of the stars at the first period, it appears better to take the mean, by giving each result a weight proportional to the number of observations of each star at the first period. The mean result so obtained is  $9'',25$ . With this result (omitting the small terms depending on  $2$  long.  $\mathcal{R}$ )

The nutation in N. P. D. =  $-8'',06 \sin(\mathcal{R} - \mathcal{Q}) - 1'',19 \sin(\mathcal{R} + \mathcal{Q})$

The nutation in  $\mathcal{R}$  =  $(-8,06 \cos(\mathcal{R} - \mathcal{Q}) - 1'',19 \cos(\mathcal{R} + \mathcal{Q})) \cot. N. P. D.$

Equation of equinoxes in  $\mathcal{R}$  =  $-15'',86 \sin \mathcal{Q}$

Equation of equinoxes in long. =  $-17'',29 \sin \mathcal{Q}$

Equation of obliquity ecliptic =  $5'',25 \cos \mathcal{Q}$

With the above nutation, the mass of the moon =  $\frac{1}{80}$ , that of the earth being unity; and the force of the moon on the sea =  $2\frac{1}{5}$ , that of the sun being unity.

Had the former observations for each star been as numerous as those in the latter, it cannot I think be doubted, that the discordances of the results would have been less. The discordance between the greatest and least result is less than one second. Hence it might perhaps be inferred, that,

supposing the constant of aberration for each star the same, two results in Table 1 should not differ nearly so much as by 1", on account of the great number of observations used in deducing the results of that Table.

The discordances between my observations and those made at Greenwich may, by some, be considered as showing the great precision of modern observations, when it is understood that the whole extent of the absolute difference between the results of the observations of the Astronomer Royal, and of those made here, is only about one second. But, independently of the interest of the question of parallax, it is highly important to ascertain the origin of this small difference. It may instruct as to the limit of accuracy actually to be attained to, when apparently there should exist no limit.

It will also appear, should any of the results that I have found be inexact, that the delicacy of an instrument cannot be appreciated by giving correctly some of the smaller motions, real or apparent, that occur, because the same instrument may, as to others, entirely mislead. Whatever may be the ultimate determinations, it is hoped, that the long and tedious exertions that have been used in obtaining these results, will not be found to have been entirely without use.

TABLE I.

	No. of Days of Observa- tion.	No. of Ob- servations, 1818-1821.	N. P. D. Jan. 1, 1819, co. lat. 86° 38' 46",5.	Const. of Aberration.	Semi-paral- lax or <i>p</i> .
Polaris - -	77	343	0 / # 1 39 24,95	" 20,18	" — 0,03
Polaris S. P.	80	337	1 39 25,16	20,12	+ 0,12
β Ursæ Majoris	75	75	32 38 59,61	20,16	+ 0,02
γ Ursæ Majoris	105	105	35 17 55,15	20,48	+ 0,39
ε - -	109	109	33 3 19,54	20,29	+ 0,33
ζ - -	94	94	34 7 34,65	20,23	+ 0,28
η Ursæ Majoris	99	99	39 46 47,18	20,76	+ 0,13
Arcturus - -	94	259	69 52 13,66	20,04	+ 0,61
β Ursæ Minoris	53	131	15 6 17,74	20,49	— 0,13
α Ophiuchi -	97	228	77 17 58,23	20,39	+ 1,57
γ Draconis -	152	152	38 29 7,51	19,86	— 0,08
α Lyræ - -	157	227	51 22 42,84	20,36	+ 1,21
α Aquilæ -	135	320	81 36 5,11	21,32	+ 1,57
α Cygni - -	94	154	45 21 42,30	20,52	+ 0,33

The heads of the respective columns sufficiently explain the numbers of this Table. It may be mentioned, that the stars near the zenith have only been observed on the meridian, and therefore the number of days of observations of these stars are the same as the number of observations. The other stars have often been observed near the meridian on each side. The stars 30° or more from the zenith have been observed twice before the reversion of the instrument, and twice after; the Pole Star occasionally still oftener. The other stars on the south side of the zenith only once in each position of the circle.

The small negative values of *p* have been put down to show the precise results; these, it is evident, may be wholly attributable to the unavoidable errors of observation; and

therefore to the extent of these quantities at least, the other results cannot be depended on.

The whole number of observations is 2633, and the mean of all the constants of aberration is 20",37.

TABLE II.

	No. of winter Observations.	Mean N. P. D. Jan. 1, 1819, by winter Observations.			N. P. D. by summer Observations.	No. of summer Observations.
		°	'	"		
$\alpha$ Ariétis - -	40	67	23	53.43	53,08	36
Aldebaran - -	65	73	51	49.35	49,09	30
$\beta$ Tauri - -	52	61	33	21.76	21,66	38
$\alpha$ Orionis - -	82	82	38	9.29	9,17	65
Castor - -	34	57	43	29.94	30,52	33
Procyon - -	64	84	19	8.44	8,41	73
Pollux -	36 Spring.	61	32	45.00	44,96	29 Autumn.
Regulus - -	21	77	9	7.45	7,58	12
$\beta$ Leonis - -	20	74	24	57.91	57,53	27

The mean of the differences of the 343 summer observations, and of the 414 winter observations, is only  $\frac{6}{100}$  of a second.



TABLE III.

	No. of Observations in summer and winter.	Zenith Distance Jan. 1, 1819, const. of Aberration for each star = $20'' \cdot 25 + x$ and $p$ = semi-parallax.	Values of $p$ .
$\alpha$ Cor. Borealis	50 S 28 W	$\begin{matrix} 0 & ' \\ 26 & 3 \end{matrix} \left\{ \begin{matrix} 23,21 + 0,29x + 0,62p \\ 24,72 - 0,24x - 0,68p \end{matrix} \right\}$	$p = 1,16 - 0,41x$
$\alpha$ Serpentis -	45 S 36 W	$46 \ 23 \left\{ \begin{matrix} 2,64 + ,09x + ,44p \\ 3,83 - ,23x - ,42p \end{matrix} \right\}$	$p = 1,35 - 0,37x$
$\delta$ Aquilæ -	37 S 27 W	$50 \ 37 \left\{ \begin{matrix} 26,61 + ,24x + ,36p \\ 28,55 + ,36x - ,25p \end{matrix} \right\}$	$p = 3,20 + 0,20x$
$\beta$ Cygni -	34 S 26 W	$25 \ 48 \left\{ \begin{matrix} 1,38 + ,30x + ,69p \\ 2,31 + ,69x - ,31p \end{matrix} \right\}$	$p = 0,93 + 0,39x$
$\gamma$ Aquilæ -	39 S 46 W	$43 \ 12 \left\{ \begin{matrix} 23,92 + ,30x + ,43p \\ 25,67 + ,38x - ,37p \end{matrix} \right\}$	$p = 2,19 + 0,10x$
$\beta$ Aquilæ -	36 S 38 W	$47 \ 25 \left\{ \begin{matrix} 25,85 + ,28x + ,37p \\ 27,50 + ,35x - ,33p \end{matrix} \right\}$	$p = 2,36 + 0,10x$
$\gamma$ Cygni -	33 S 26 W	$13 \ 42 \left\{ \begin{matrix} 15,41 + ,55x + ,64p \\ 16,40 + ,58x - ,60p \end{matrix} \right\}$	$p = 0,80 + 0,02x$
$\epsilon$ Delph. -	28 S 17 W	$42 \ 41 \left\{ \begin{matrix} 29,84 + ,41x + ,33p \\ 31,12 + ,36x - ,40p \end{matrix} \right\}$	$p = 1,75 - 0,07x$
$\alpha$ - - -	11 S 9 W	$38 \ 6 \left\{ \begin{matrix} 23,48 + ,41x + ,42p \\ 24,86 + ,42x - ,42p \end{matrix} \right\}$	$p = 1,64 + 0,01x$
$\lambda$ Cygni - -	14 S 10 W	$17 \ 33 \left\{ \begin{matrix} 21,53 + ,49x + ,63p \\ 23,01 + ,65x - ,49p \end{matrix} \right\}$	$p = 1,32 + 0,14x$
$\nu$ - - -	34 S 18 W	$12 \ 54 \left\{ \begin{matrix} 42,62 + ,51x + ,67p \\ 43,24 + ,73x - ,44p \end{matrix} \right\}$	$p = 0,56 + 0,20x$
$\gamma$ Equulei -	12 S 9 W	$43 \ 58 \left\{ \begin{matrix} 40,35 + ,41x + ,29p \\ 42,34 + ,34x - ,38p \end{matrix} \right\}$	$p = 2,97 - 0,10x$
$\alpha$ - - -	10 S 10 W	$48 \ 52 \left\{ \begin{matrix} 51,82 + ,39x + ,19p \\ 53,39 + ,26x - ,37p \end{matrix} \right\}$	$p = 2,80 - 0,23x$
$\epsilon$ Pegasi -	26 S 21 W	$44 \ 20 \left\{ \begin{matrix} 10,80 + ,42x + ,25p \\ 11,99 + ,19x - ,45p \end{matrix} \right\}$	$p = 1,70 - 0,33x$
$\alpha$ Aquarii -	36 S 30 W	$54 \ 34 \left\{ \begin{matrix} 52,55 + ,37x + ,11p \\ 53,25 + ,01x - ,39p \end{matrix} \right\}$	
55 l Pegasi -	33 S 45 W	$44 \ 57 \left\{ \begin{matrix} 9,64 + ,41x + ,15p \\ 9,94 + ,16x - ,39p \end{matrix} \right\}$	

The above results will appear very extraordinary; and although they are explained by parallax, yet many circumstances of these stars, both as to magnitude and position, will much weaken that explanation, and, on the whole, the results may be thought to have increased the difficulties of the subject. It is evident, that, for most of these stars, the terms depending on  $x$  can have little influence, considering the smallness of the coefficients, and the probable small values of  $x$ .

TABLE IV.  $\eta$  Ursæ Majoris.

Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.
1818.			1819.					
June 17	I W	-0,65	Oct. 27	I W	+0,41	May 19	I E	-0,14
20	I E	+0,83	29	I W	+0,66	22	I E	+0,13
22	I W	-0,07	Nov. 1	I E	+0,53	24	I W	-1,04
28	I E	+0,42	2	I W	+0,66	June 23	I W	-0,24
29	I W	+0,39	5	I E	-0,62	24	I E	+0,21
July 2	I E	-0,56	7	I W	-0,06	25	I W	-1,55
5	I W	+1,39	15	I E	+0,19	26	I E	-0,80
15	I E	+0,39	20	I W	-0,91	27	I E	-0,85
Nov. 2	I E	-0,09	21	I E	+0,09	28	I W	+0,33
6	I W	+0,88	23	I W	-0,07	July 6	I W	+0,64
9	I E	+1,51	25	I E	-0,11	7	I E	+0,41
14	I W	+0,47	Dec. 2	I W	+0,23	8	I W	-0,67
22	I E	-1,19	15	I W	-0,79	10	I E	-0,63
23	I W	+0,45	24	I E	+1,38	13	I W	+0,03
28	I E	-1,18	30	I W	+0,51	15	I E	+0,17
Dec. 1	I E	+0,09	1820			Oct. 4	I W	+0,36
2	I W	-1,48	Jan. 10	I E	-1,19	27	I E	+0,04
4	I E	+0,21	April 14	I E	-0,94	28	I W	+0,29
8	I W	+2,47	18	I W	-0,14	29	I E	-0,14
15	I E	+1,66	21	I E	+2,76	31	I W	+0,35
17	I W	+0,87	22	I W	+1,35	Nov. 1	I E	-0,36
1819			24	I E	+0,04	2	I E	+0,36
June 24	I W	-0,39	25	I W	+0,18	16	I W	-0,16
July 3	I E	-0,43	30	I E	+1,09	17	I E	-0,92
4	I W	+0,81	May 1	I W	-0,14	21	I W	-1,82
8	I E	+0,88	3	I E	-0,78	23	I E	-0,72
14	I W	+0,84	4	I W	+0,01	Dec. 10	I W	-0,74
20	I E	-1,01	6	I E	-0,75	18	I W	-0,58
21	I E	-0,66	8	I W	-0,23	1821		
24	I W	-0,33	10	I W	-1,52	Jan. 2	I E	-0,27
27	I E	+0,87	12	I E	+0,11	3	I W	-1,07
29	I W	-1,25	13	I E	-0,68	16	I W	+0,46
31	I W	-0,30	17	I W	+0,99	19	I E	-0,45
Oct. 15	I E	+0,31	18	I W	+1,39	26	I W	-0,44

TABLE V.  $\gamma$  Draconis.

Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.
1817.			1819.			1820.		
Nov. 11	I W	-0,17	Jan. 31	I W	-0,06	March 2	I E	-0,64
16	I E	+1,74	Feb. 1	I E	-1,05	8	I W	-0,64
17	I W	-2,01	July 14	I E	+0,99	June 28	I E	+0,32
Dec. 6	I W	+0,23	15	I W	-0,22	July 5	I W	+1,66
10	I W	+0,76	20	I E	+0,21	7	I W	+0,66
21	I W	+1,80	21	I W	+0,31	8	I E	+0,30
22	I E	-0,22	24	I E	-1,07	10	I W	+0,78
23	I E	-0,41	27	I W	-1,02	13	I E	+0,69
24	I W	+0,37	28	I W	-1,61	15	I W	-0,46
26	I E	-0,88	29	I E	+0,46	18	I E	+0,60
27	I E	+0,20	30	I W	+0,33	19	I W	-0,83
30	I W	-0,06	31	I E	+1,28	21	I E	-1,41
1818.			Aug. 2	I W	+0,85	24	I W	-0,51
Jan. 5	I E	-1,45	4	I E	-1,26	28	I E	0,00
7	I W	-0,75	7	I W	+0,86	Aug. 1	I W	-1,41
20	I E	-0,17	8	I E	-1,71	31	I E	+0,68
23	I W	-0,88	9	I E	+0,36	Sept. 1	I W	+0,49
30	I E	-0,38	12	I W	-1,27	2	I W	+2,43
Feb. 1	I E	-1,48	15	I E	+0,95	6	I E	+1,09
July 19	I W	+0,38	18	I W	-1,31	8	I E	-0,08
25	I E	+0,35	19	I E	-1,11	15	I W	+0,18
27	I W	+0,70	20	I W	+1,12	16	I W	-0,06
Aug. 1	I E	+1,97	21	I E	-0,10	17	I E	+1,24
2	I W	-0,37	22	I W	+0,81	18	I E	+0,47
6	I E	-1,61	Sept. 10	I W	+2,45	20	I W	+0,34
7	I E	+0,80	14	I E	-0,96	Oct. 4	I E	-1,25
10	I E	-0,93	16	I W	+0,21	17	I E	+0,73
11	I W	+1,61	20	I E	+0,17	25	I W	+0,21
12	I E	-0,55	22	I E	-0,14	Nov. 2	I W	-0,27
13	I W	-0,34	23	I W	+0,70	4	I E	-0,76
14	I E	-0,75	27	I E	-1,22	10	I W	-1,03
15	I W	-2,94	Oct. 2	I W	-1,72	Dec. 19	I W	-0,77
16	I E	+1,20	3	I E	+0,16	27	I E	-0,71
Nov. 24	I W	+0,49	4	I W	-0,16	1821.		
Dec. 5	I E	-1,26	20	I W	-0,34	Jan. 2	I E	+1,38
7	I W	-0,44	24	I E	+0,98	13	I W	-1,46
9	I E	-0,99	26	I W	-0,47	19	I W	-0,61
15	I W	-0,12	29	I E	+0,66	Feb. 1	I E	-0,72
28	I W	-1,72	30	I W	-0,01	4	I E	+0,36
29	I E	+3,00	Nov. 8	I E	+1,10	6	I W	-0,48
1819.			Dec. 13	I W	+0,27	7	I W	-0,31
Jan. 8	I W	+1,64	14	I E	-0,15	8	I E	-0,41
10	I E	-0,79	15	I W	-1,04	10	I E	-1,75
11	I W	+0,39	22	I E	+1,62	14	I W	+0,78
12	I E	+1,22	24	I E	+0,77	19	I W	+0,50
17	I W	+0,72	25	I W	+1,43	24	I E	-0,47
18	I E	+0,42	27	I E	+0,64	27	I W	+0,18
19	I W	-1,08	28	I W	+0,50	March 9	I W	+1,35
20	I E	-1,14	1820.			11	I E	-0,87
22	I W	+0,44	Jan. 3	I W	+1,75	12	I W	-1,73
25	I E	-0,31	15	I E	+1,12	13	I E	+1,71
29	I W	-1,61	27	I E	-0,18	15	I W	+1,02

TABLE VI.  $\alpha$  Lyrae.

Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.
1818.			1819.			1820.		
July 14	1 W	-0,77	Feb. 24	1 W	+0,21	Mar. 22	1 E 1 W	-0,41
15	1 E	-0,26	July 3	1 E	-2,80	24	1 E 1 W	-0,13
16	1 W	-0,18	4	1 E	-0,06	25	1 E 1 W	+0,84
17	1 W	-2,67	14	1 E	-1,12	April 5	1 W 1 E	-0,64
19	1 E	-1,65	15	1 W	+0,32	6	1 E 1 W	-0,73
24	1 E	+0,41	20	1 E	+0,16	July 5	1 W 1 E	+1,78
25	1 E	-2,35	21	1 W	+0,59	7	1 W 1 E	+0,30
27	1 W	-1,37	24	1 E	+0,20	8	1 E 1 W	+0,44
Aug. 1	1 E	+1,13	28	1 W	-1,63	10	1 W 1 E	+0,82
2	1 E	-0,21	29	1 E	-0,42	13	1 E 1 W	+0,52
6	1 W	-1,66	30	1 W	-1,32	15	1 W 1 E	-0,48
7	1 W	+1,50	Aug. 2	1 W	-0,71	18	1 E 1 W	+1,55
9	1 W	+0,70	4	1 E	-0,77	19	1 W 1 E	+1,76
10	1 E	-0,39	7	1 W	-1,28	24	1 W 1 E	+0,84
11	1 E	-0,28	9	1 E	+0,10	25	1 E 1 W	+0,77
12	1 E	+0,43	15	1 W	-3,08	Sept. 12	1 W 1 E	+0,96
13	1 W	-0,55	18	1 E	+0,28	15	1 W 1 E	+0,17
14	1 E	+0,07	19	1 W	+0,01	18	1 E 1 W	+0,48
15	1 W	-0,64	21	1 E	-0,72	20	1 W 1 E	-0,06
16	1 E	-0,34	23	1 W	-0,60	Oct. 4	1 E 1 W	+0,47
Oct. 16	1 W	+0,96	27	1 E	-2,69	5	1 E 1 W	+1,15
17	1 E	+1,98	31	1 W	+1,38	18	1 W 1 E	+0,82
19	1 W	-0,72	Sept. 8	1 W	+1,55	25	1 W 1 E	+0,27
20	1 E	-0,91	10	1 W	+1,34	Nov. 1	1 W 1 E	-0,17
26	1 E	+1,12	12	1 E	-0,93	2	1 W 1 E	-0,81
Nov. 2	1 W	+0,23	14	1 E	-2,85	Dec. 2	1 W 1 E	+0,39
3	1 E	-1,22	16	1 W	-0,32	11	1 W	-0,07
7	1 E	-0,62	20	1 E 1 W	+0,82	19	1 W 1 E	+0,44
8	1 W	-0,19	21	1 E	-0,99	23	1 E 1 W	-0,86
24	1 W	+1,16	22	1 W	-1,05	28	1 E 1 W	+0,30
Dec. 5	1 E	-0,97	23	1 W	-0,91	1821.		
7	1 W	+0,75	Oct. 2	1 E 1 W	-0,46	Jan. 2	1 E 1 W	+0,45
9	1 E	+0,01	4	1 E 1 W	+0,63	13	1 W 1 E	+0,33
15	1 W	-2,44	17	1 E	+1,73	19	1 W 1 E	-0,20
16	1 E	+0,32	20	1 E 1 W	+0,25	Feb. 1	1 E 1 W	-1,45
21	1 W	-3,41	29	1 E	+1,68	2	1 W 1 E	-0,85
22	1 W	-1,38	30	1 W	+0,20	6	1 W 1 E	-0,58
1819.			Nov. 2	1 E	+1,20	8	1 E 1 W	+0,31
Jan. 8	1 W	+1,22	8	1 W	-0,87	10	1 E 1 W	-0,06
10	1 E	-1,58	Dec. 13	1 W 1 E	-0,61	14	1 W 1 E	+1,42
11	1 W	-1,40	15	1 W	-1,95	19	1 W 1 E	-1,36
12	1 E	-0,88	16	1 E 2 W	-1,19	24	1 E 1 W	+1,10
17	1 W	-0,05	23	1 E	+1,06	27	1 W 1 E	-0,62
18	1 E	-1,30	26	2 W 2 E	+0,92	Mar. 9	1 W 1 E	+0,25
19	1 W	+0,53	29	1 W 1 E	+1,05	10	1 W 1 E	-0,03
20	1 E	-0,91	30	1 W 1 E	+1,25	11	1 E 1 W	-0,23
25	1 E	-1,20	1820.			12	1 W 1 E	-0,54
29	1 W	+0,18	Jan. 2	1 E 1 W	+0,92	13	1 E 1 W	+1,08
31	1 E	-0,93	5	1 W 1 E	+0,36	15	1 W 1 E	+1,23
Feb. 1	1 E	-1,58	15	1 E 1 W	+0,88	19	1 W 1 E	+1,26
5	1 W	-0,40	16	1 W 1 E	+0,50	21	1 E 1 W	+1,56
6	1 W	-0,08	Mar. 2	1 E 1 W	+0,28	22	1 W 1 E	+0,44
9	1 E	-0,16	8	1 W 1 E	+0,13			
21	1 E	-0,90	19	1 E 1 W	-0,81			

TABLE VII.  $\alpha$  Aquilæ.

Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.	Date of Observation.	Obs. Face, East or West.	Error of Observation.
1818.			1819.			1820.		
July 14	1 W	-0,50	Feb. 12	1 W	-1,17	Mar. 19	2 E 2 W	-1,09
17	1 E	-1,33	13	1 E	+0,37	20	2 E —	-0,31
24	1 W	-1,85	15	1 W	-1,08	22	— 2 W	-0,79
25	1 E	-1,89	21	1 E	+0,02	24	2 E 2 W	+0,33
Aug. 1	1 E	-0,72	22	1 W	+0,27	28	2 E 2 W	+0,15
2	1 W	-2,12	23	1 E	-0,73	29	2 E 2 W	-0,44
6	1 E	-3,88	24	1 W	-0,01	April 5	2 E —	-1,38
9	1 W	-1,65	Mar. 3	1 E	+2,24	6	— 2 W	-2,04
10	1 E	-1,66	Aug. 4	1 E	-2,65	14	2 E 2 W	+0,55
11	1 W	-1,86	7	1 W	-1,47	July 28	2 E 2 W	-0,12
12	1 E	-1,60	9	1 E	-1,60	31	2 E 2 W	+1,01
13	1 W	-1,95	11	1 W	-2,48	Aug. 1	2 E 2 W	+0,08
14	1 E	-0,57	12	1 E	+0,48	4	2 E 2 W	+0,50
15	1 W	-0,42	15	1 W	-1,65	5	2 E 2 W	+1,10
16	1 E	-0,06	18	1 E	+0,32	9	2 E 2 W	+0,47
Oct. 16	1 W	-0,62	20	1 W	-1,14	10	2 E 2 W	+0,58
17	1 E	-0,52	21	1 E	-0,34	17	2 E 2 W	-0,94
21	1 W	-0,05	22	1 W	+0,14	18	2 E 2 W	+0,06
Nov. 1	1 E	-0,07	24	1 E	+1,63	19	2 E 2 W	-0,05
2	1 W	-1,22	27	1 E	+1,78	Sept. 15	2 E 2 W	+0,16
3	1 E	-1,90	Sept. 1	1 W	+2,21	17	2 E 2 W	-0,34
5	1 W	-0,44	3	1 E	-1,74	18	2 E 2 W	-0,97
7	1 E	+0,30	6	1 W	+2,96	20	2 E 2 W	+0,19
8	1 W	+0,65	7	1 E	+0,29	Oct. 4	2 E 2 W	+1,63
14	1 E	+0,22	8	2 E 2 W	+0,12	17	2 E 2 W	+0,36
20	1 W	+0,30	10	2 E 2 W	-0,39	18	2 E 2 W	+1,75
23	1 E	-0,11	11	2 E 2 W	+0,49	4	2 E —	+1,18
24	1 W	-0,51	12	2 E 2 W	+0,38	25	2 E 2 W	+0,06
Dec. 7	1 E	+1,46	14	2 E 2 W	-0,45	28	2 E 2 W	+0,83
8	1 W	-2,24	16	2 E 2 W	-0,12	Dec. 2	2 E 2 W	-0,27
9	1 W	+1,85	19	2 E 2 W	+0,36	23	2 E 2 W	-0,71
15	1 E	-0,68	20	2 E 2 W	+0,07	28	2 E 2 W	+0,47
16	1 W	-0,26	21	2 E 2 W	+1,89	1821.		
18	1 E	+1,03	Dec. 3	2 E 1 W	-1,03	Jan. 19	2 E 2 W	-1,31
21	1 W	-1,75	8	2 E 2 W	+0,14	Feb. 1	2 E 2 W	-1,13
22	1 E	-0,01	13	2 E 2 W	+0,21	9	2 E 2 W	-0,27
29	1 E	+1,68	14	2 E 2 W	-0,64	10	2 E 2 W	-0,01
30	1 W	-0,69	15	2 E 2 W	-0,59	19	2 E 2 W	+0,33
1819			23	2 E 2 W	-0,34	24	2 E 2 W	+0,57
Jan. 2	1 E	+0,65	26	2 E 2 W	-0,08	27	2 E 2 W	+1,13
19	1 W	-0,98	29	2 E 2 W	-0,63	Mar. 3	2 E 2 W	+1,38
20	1 E	-4,16	1820.			9	2 E 2 W	+0,46
22	1 W	-0,54	Jan. 3	— 2 W	-0,48	10	2 E 2 W	+0,12
Feb. 1	1 E	-1,08	6	2 E 2 W	-0,42	11	2 E 2 W	+0,27
6	1 W	-0,79	Mar. 8	2 E 2 W	+1,03	13	2 E 2 W	+0,86
9	1 E	-0,92	16	2 E 2 W	+1,63	15	2 E 2 W	+1,39

In the above, it might have been better to have omitted the observations of August 6, 1818, and January 20, 1819. No difference, however, would have taken place in the total results; and it may be proper here to remark, that no result has been omitted, except some error is clearly shown by the circumstances of the observation; and this has not happened in any thing like ten instances in above 4000 observations.

TABLE VIII.  $\alpha$  Aquarii.

	Face of Circle.	Time from Meridian.	Bottom Microscope.	Refraction.	Mean Z. D. Jan. 1, 1819.		Face of Circle.	Time from Meridian.	Bottom Microscope.	Refraction.	Mean Z. D. Jan. 1, 1819.	
1818.							1818.					
Sept. 2	E	7 38,2	54 35 43,8	1 20,22	54 34 53,32		Dec. 28	E	3 27,2	54 34 39,2	1 24,07	54 34 53,91
	E	5 50,7	35 9,4	1 20,20	53,61	+ ,37 x		E	1 0,2	34 23,6	1 24,07	53,99
	E	3 48,2	34 41,0	1 20,18	53,49		In.	W	6 50,8	34 0,3	1 24,11	53,13
In.	W	4 46,8	33 29,6	1 20,18	52,82	+ ,12 p	Bar. 30,37	W	9 3,8	34 49,5	1 24,15	51,72
Bar. 29,59	W	6 36,3	33 59,2	1 20,20	52,59		Therm.	W	12 8,8	36 22,3	1 24,23	50,60
Therm.							Int. 43 $\frac{1}{2}$					
Int. 53 $\frac{1}{2}$					53,09		Ext. 38 $\frac{1}{2}$					52,67
Ext. 49 $\frac{1}{2}$					53,15							
4	E	7 52,7	35 47,7	1 19,52	51,25		1819.					
	E	5 49,7	35 8,6	1 19,48	52,48	+ ,37 x	Aug. 17	W	11 40,2	35 51,5	1 19,83	49,62
In.	W	3 12,8	33 12,6	1 19,47	53,22	+ ,11 p		W	8 18,2	34 15,0	1 19,77	49,26
Bar. 29,60	W	5 24,8	33 38,2	1 19,48	51,56			W	6 27,2	33 36,7	1 19,72	49,88
Therm.	W	7 23,8	34 15,8	1 19,52	52,70			E	4 22,2	33 6,1	1 19,72	51,67
Int. 58 $\frac{1}{2}$							In.	W	6 10,8	34 56,7	1 19,72	51,44
Ext. 53 $\frac{1}{2}$					52,24		Bar. 30,13	E	8 19,8	35 41,5	1 19,77	51,66
							Therm.	E	9 58,8	36 23,9	1 19,80	50,89
							Int. 65 $\frac{1}{2}$	E	11 49,8	37 23,5	1 19,83	52,82
							Ext. 57 $\frac{1}{2}$					50,90
Dec. 16	W	8 30,8	34 34,9	1 23,53	50,65							
	W	6 51,8	33 58,4	1 23,51	50,55	+ ,02 x						
	W	5 6,8	33 28,9	1 23,48	51,14							
In.	E	3 33,2	34 38,4	1 23,48	51,76	- ,39 p	18	W	7 59,9	34 10,5	1 19,89	52,01
Bar. 29,87	E	5 28,2	35 3,7	1 23,51	52,24			W	4 57,9	33 16,3	1 19,83	54,10
Therm.	E	7 6,2	35 31,5	1 23,53	50,56			W	3 2,9	32 52,4	1 19,81	52,25
Int. 39 $\frac{1}{2}$							In.	E	6 18,1	34 59,2	1 19,83	52,07
Ext. 36 $\frac{1}{2}$					51,15		Bar. 30,06	E	8 17,1	35 45,8	1 19,89	52,23
							Therm.	E	10 12,1	36 32,0	1 19,93	52,82
							Int. 65 $\frac{1}{2}$					52,58
							Ext. 59 $\frac{1}{2}$					
18	E	11 38,9	37 34,7	1 23,36	51,24							
	E	7 49,9	35 47,9	1 23,26	51,12	+ ,01 x						
	E	4 56,9	34 55,7	1 23,22	51,82							
In.	W	4 58,1	33 28,7	1 23,28	52,90	- ,39 p	20	E	9 48,0	36 20,6	1 19,67	52,49
Bar. 29,80	W	8 57,1	34 48,0	1 23,36	52,64			E	7 19,0	35 18,8	1 19,62	51,73
Therm.	W	12 9,1	36 26,2	1 23,43	53,90			E	4 54,0	34 37,1	1 19,58	52,29
Int. 39 $\frac{1}{2}$							In.	W	6 59,0	33 49,9	1 19,58	53,04
Ext. 35 $\frac{1}{2}$					52,27		Bar. 30,01	W	9 18,0	34 43,1	1 19,62	52,04
							Therm.	W	12 37,0	36 28,8	1 19,72	53,44
							Int. 63 $\frac{1}{2}$					52,50
							Ext. 58 $\frac{1}{2}$					
21	W	10 16,2	35 24,7	1 23,30	52,96							
	W	7 9,2	34 8,0	1 23,24	54,19	- ,01 x						
	W	4 41,2	33 24,1	1 23,18	52,18							
In.	E	5 47,8	35 10,2	1 23,20	53,28	- ,39 p	21	E	10 12,0	35 6,1	1 19,62	49,95
Bar. 30,01	E	8 16,8	36 0,0	1 23,26	52,92			E	8 13,0	34 14,3	1 19,56	50,59
Therm.	E	10 29,8	36 58,7	1 23,30	51,86			E	6 8,0	33 31,1	1 19,46	50,24
Int. 43 $\frac{1}{2}$								E	4 17,0	33 3,9	1 19,46	50,73
Ext. 40 $\frac{1}{2}$					52,90			W	5 0,0	34 39,4	1 19,51	53,32
							In.	W	7 4,0	35 14,5	1 19,56	52,64
							Bar. 29,96	W	8 48,0	35 53,4	1 19,56	51,99
							Therm.	W	11 58,0	37 26,2	1 19,67	51,26
							Int. 63 $\frac{1}{2}$					51,34
							Ext. 57 $\frac{1}{2}$					
22	E	10 16,6	35 53,0	1 23,72	53,21							
	E	7 2,6	35 32,5	1 23,65	53,07	- ,02 x						
	E	3 25,6	34 38,3	1 23,61	53,21							
In.	W	5 43,4	33 40,7	1 23,63	53,76	- ,39 p	22	E	8 50,4	35 55,5	1 19,46	53,10
Bar. 30,13	W	8 19,4	34 34,2	1 23,67	54,83			E	7 12,4	35 17,4	1 19,46	53,00
Therm.	W	11 53,4	36 16,6	1 23,76	53,75			E	5 26,4	34 45,7	1 19,41	52,03
Int. 42 $\frac{1}{2}$								E	3 23,4	34 20,9	1 19,41	54,24
Ext. 40 $\frac{1}{2}$					53,64			W	7 59,6	34 11,5	1 19,46	53,00
							In.	W	9 49,6	34 57,8	1 19,51	52,45
							Bar. 29,95	W	12 13,6	36 12,5	1 19,56	51,10
							Therm.	W	14 35,6	37 43,8	1 19,61	51,25
							Int. 64 $\frac{1}{2}$					52,65
							Ext. 58 $\frac{1}{2}$					
24	W	3 6,1	33 8,6	1 22,12	53,25							
	W	0 40,1	32 54,4	1 22,10	52,21	- ,03 x						
	W	0 49,9	32 55,0	1 22,10	52,46							
In.	E	9 43,9	36 37,1	1 22,15	51,20	- ,39 p						
Bar. 29,82	E	11 8,9	37 21,1	1 22,27	52,82							
Therm.	E	13 9,9	38 33,2	1 22,31	54,56							
Int. 46 $\frac{1}{2}$												
Ext. 44 $\frac{1}{2}$					52,75							



TABLE VIII. continued.

	Face of Circle.	Time from Meridian.	Bottom Microscope.	Refraction.	Mean Z. D. Jan. 1, 1819.	
1820. Sept. 8  In. Bar. 30,05 Therm. Int. 58° Ext. 54°	E	5 45,5	54 34 31,0	1 20,65	54 34 52,92	+ ,38 x + ,09 p
	E	4 10,0	34 7,6		52,22	
	E	2 27,0	33 51,4		52,35	
	W	4 47,0	32 48,5		53,11	
	W	7 13,0	33 29,0		51,65	
	W	8 44,0	34 4,5		52,39	
					52,44	
					52,44	
10  In. Bar. 30,01 Therm. Int. 61° <sup>1</sup> / <sub>8</sub> Ext. 55° <sup>1</sup> / <sub>2</sub>	E	2 47,4	33 54,3	1 19,99	52,14	+ ,38 x + ,07 p
	E	0 42,4	33 43,8		52,10	
	E	0 41,6	33 42,8		51,13	
	W	5 50,6	33 4,3		52,18	
	W	7 39,6	33 39,4		52,03	
	W	10 47,1	35 1,0		50,81	
					51,73	
					51,73	
11  In. Bar. 29,87 Therm. Int. 62° Ext. 58°	W	6 11,4	33 12,0	1 19,51	53,46	+ ,39 x + ,06 p
	W	5 1,4	32 53,3		53,56	
	W	3 23,4	32 33,2		53,21	
	E	3 13,6	34 0,7		54,34	
	E	4 34,1	34 14,1		52,71	
	E	6 21,6	34 42,7		53,18	
					53,41	
					53,41	
17  In. Bar. 29,61 Therm. Int. 53° Ext. 44°	E	8 14,4	35 19,8	1 20,29	51,91	+ ,39 x + ,02 p
	E	7 16,4	34 58,7		52,36	
	E	5 34,4	34 26,4		51,44	
	W	0 12,0	32 14,0		51,51	
	W	2 1,6	32 20,3		51,99	
	W	3 28,1	32 31,3		51,61	
					51,80	
					51,80	
Dec. 19  In. Bar. 29,94 Therm. Int. 46° Ext. 44°	E	5 18,2	34 25,0	1 22,42	53,67	— ,00 x — ,39 p
	E	4 5,2	34 8,3		53,39	
	E	3 7,2	33 58,3		53,40	
	W	3 27,8	32 35,0		55,17	
	W	4 52,8	32 51,3		54,49	
	W	5 53,8	33 3,8		51,24	
					53,56	
					53,56	
28  In. Bar. 29,72 Therm. Int. 35° Ext. 33°	E	8 45,4	35 36,2	1 23,78	55,89	— ,06 x — ,39 p
	E	2 25,4	33 53,8		55,20	
	E	0 36,4	33 44,5		53,81	
	W	4 57,6	32 52,3		55,12	
	W	6 49,6	33 23,6		54,81	
	W	8 27,6	33 59,2		54,55	
					54,90	
					54,90	

To show the consistency of the several observations of each day, they have been reduced, which otherwise would have been unnecessary, by applying the correction for collimation to each reading off. These corrections are for the *bottom* microscope, which was only used for  $\alpha$  Aquarii. They were as follow:

1818.	Sept. 2, 4	41,39	} — face East.
	Dec. 16, 28	44,15	
1819.	Aug. 17—Sept. 16	41,70	} + face West.
	Dec. 3, 28	42,61	
1820.	Sept. 1, 17	43,96	}
	Dec. 19, 28	44,16	

Corrections of collimation for  $\gamma$  and  $\beta$  Aquilæ for mean of three microscopes.

$\gamma$  Aquilæ.

1818.	July 14—Aug. 16	54,81	} — face East.
	Oct. 16—Nov. 14	47,90	
	Nov. 20—Nov. 24	41,12	} + face West.
	Dec. 7, 9	43,43	
1819.	Aug. 2—Sept. 7	47,04	}
	Nov. 16, 29	47,25	
1820.	July 13, 25	48,20	}
	Oct. 29 — Nov. 21	48,40	

$\beta$  Aquilæ.

1818.	July 17—Aug. 16	55,24	} — face East.
	Oct. 16—Nov. 14	48,22	
	Nov. 20—Nov. 24	41,34	} + face West.
1819.	Aug. 2—Sept. 7	47,10	
	Nov. 16, 26	47,51	}
1820.	July 13, 25	48,53	
	Oct. 29 — Nov. 21	48,28	





TABLE X.  $\beta$  Aquilæ.

		Face of Circle.	Mean of three Microscopes.	Refraction.	Mean Z. D. Jan. 1, 1819.	$x$	$\rho$			Face of Circle.	Mean of three Microscopes.	Refraction.	Mean Z. D. Jan. 1, 1819.	$x$	$\rho$
1818.								1818.							
July	17	E	47 25 29,83	1 1,14	47 25 26,64	+15	+46	Oct.	16	W	47 23 37,93	1 1,94	47 25 26,24	+46	-14
	24	W	23 37,53	1 0,48	25,31	,21	,44		17	E	25 13,17	1 2,22	25,29	,46	,15
	25	E	25 24,77	1 1,33	23,09	,21	,43		21	W	23 36,73	1 2,25	25,21	,45	,18
Aug.	6	E	25 22,13	1 1,95	22,83	,29	,39	Nov.	1	E	25 16,73	1 2,17	28,20	,41	,26
	9	W	23 32,53	1 2,33	24,48	,31	,37		2	W	23 38,53	1 1,48	25,70	,40	,26
	10	E	25 23,90	1 2,23	25,41	,32	,36		3	E	25 14,57	1 1,35	25,11	,40	,27
	11	W	23 32,73	1 2,43	25,04	,33	,36		7	E	25 15,17	1 2,43	26,50	,38	,30
	12	E	25 22,40	1 2,30	24,24	,33	,35		8	W	23 39,97	1 2,78	28,02	,38	,31
	13	W	23 32,97	1 2,36	25,47	,34	,35		14	E	25 16,63	1 1,36	26,35	,34	,35
	14	E	25 24,43	1 2,29	26,49	,35	,34		20	W	23 48,57	1 1,95	27,88	,29	,39
	16	E	25 24,63	1 2,22	26,84	,36	,33		23	E	25 11,53	1 1,39	27,30	,27	,40
									24	W	23 47,40	1 2,36	26,71	,27	,41
1819.								1819.							
Aug.	2	W	23 33,80	1 0,92	23,60	,26	,40				Time from Meridian	Mean of three Microscopes.	Refraction.	Mean Z. D. Jan. 1, 1819.	
	4	E	25 9,33	1 1,30	25,58	,28	,40	Nov.	16	E	0 34,5	47 25 6,53	1 3,59	47 25 27,20	+33 x
	9	E	25 8,30	1 1,46	25,40	,31	,37		W	6 16,5	24 35,27	1 3,63	29,23	-36 p	
	11	W	23 33,20	1 0,98	24,29	,33	,36		22	E	2 23,7	25 17,27	1 3,95	29,22	+28 x
	15	W	23 33,23	1 1,46	25,29	,35	,34		W	8 28,7	25 25,60	1 3,98	27,95	-40 p	
	18	E	25 8,47	1 1,45	26,65	,37	,32		23	W	0 0,0	23 31,70	1 4,62	28,30	+27 x
	20	W	23 32,97	1 1,23	25,37	,38	,30		E	12 39,4	29 18,80	1 4,82	27,29	-40 p	
	21	E	25 8,37	1 1,40	26,85	,39	,29		24	E	1 56,0	25 11,90	1 5,15	27,98	+27 x
Sept.	22	W	23 31,60	1 1,25	24,25	,39	,29		W	6 32,0	24 36,00	1 5,19	25,52	-41 p	
	1	W	23 34,70	1 1,87	28,94	,43	,22		26	W	1 1,4	23 33,60	1 4,13	27,70	+25 x
	3	E	25 4,00	1 1,57	23,92	,44	,21		E	8 12,6	26 52,80	1 4,21	26,99	-41 p	
	6	W	23 34,10	1 1,72	28,58	,45	,19		1820.						
	7	E	25 6,57	1 0,95	26,15	,45	,18	Oct.	29	E	4 22,5	25 28,07	1 1,75	27,13	+42 x
								W	4 54,0	23 59,60			27,52	-24 p	
								Nov.	1	W	1 53,8	23 26,60	1 2,93	27,86	+41 x
								E	5 10,2	25 38,83			26,94	-26 p	
								2	W	0 48,3	23 21,73	1 3,12	27,81	+40 x	
								E	6 15,2	25 57,80			26,49	-26 p	
								7	W	0 48,5	23 23,27	1 1,95	27,84	+38 x	
								E	4 37,5	25 33,63			28,84	-30 p	
								11	E	0 51,4	24 57,77	1 4,13	27,58	+36 x	
								W	7 50,4	24 56,53			26,83	-33 p	
								15	W	2 57,9	23 34,40	1 4,37	27,93	+34 x	
								E	2 21,1	25 6,20			28,33	-35 p	
								17	E	1 58,7	25 6,37	1 2,90	29,90	+32 x	
								W	5 31,3	24 11,83			29,38	-36 p	
								21	E	2 32,5	25 12,07	1 1,45	29,27	+29 x	
								W	3 10,5	23 41,27			29,30	-39 p	